MATHS WORKSHOPS

Algebra, Linear Functions and Series

Business School

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Outline

Outline

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Variables & Parameters

$$5x + 2 = 12$$

$$ax + b = c$$

Definition (Parameters)

A parameter is some fixed value, also known as a "constant" or "coefficient." They are generally given letters from the start of the alphabet. In the above equations, 5, 2, 12, a, b and c are the parameters.

More

Definition (Variables)

A variable is an unknown value that may change, or vary, depending on the parameter values. Variables are usually denoted by letters from the end of the alphabet. In the above equations x is the variable.

More

Basics of algebraic mathematics

Definition (Algebraic variables)

A variable is an unknown number that is usually represented by a letter of the alphabet. Like numbers, they can be added, subtracted, multiplied and divided.

$$w + w = 2w$$
$$3x - 2x = x$$
$$y \times y = y^2$$
$$2z \div z = \frac{2z}{z} = 1$$

Note how each different variable (different letter of the alphabet) corresponds to a different number. Same variables represent the same unknown number and that's why they can be added and subtracted with like variables.

More

Solving for a particular variable

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Definition (Solving an equation)

We can solve an equation by using mathematical operations (addition, subtraction, multiplication and division) to rearrange the equation such that the variable is on one side of the equation and the parameters are all on the other side. More

Solve for x:

$$ax + b = c$$

$$ax = c - b \qquad \text{(subtracting } b \text{ from both sides)}$$

$$x = \frac{c - b}{a} \qquad \text{(dividing both sides by } a\text{)}$$

We have the variable, x, on the left hand side and all the parameters, a, b and c, on the right hand side.

How does a variable vary?

Our solution is:

$$x = \frac{c-b}{a}$$

If we change the values of the parameters, this will change the value of variable, x. I.e. x varies according to the choice of the (fixed) parameters.

Example (Substitute: $a = 2$, $b = 3$, $c = 4$)

$$x = \frac{c-b}{a} = \frac{4-3}{2} = \frac{1}{2} = 0.5.$$

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Example (Try yourself by substituting: $a = 5$, $b = 1$, $c = 2$)

$$x = \frac{c-b}{a} =$$

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Example (Try yourself by substituting: $a = 5$, $b = 1$, $c = 2$)

$$x = \frac{c-b}{a} = \frac{2-1}{5} = \frac{1}{5} = 0.2.$$

Your turn...

1.

$$x + 7 = 12$$

2.

$$\frac{x}{5} = 6$$

Your turn. . .

1.

$$\begin{aligned} x + 7 &= 12 \\ x + 7 - 7 &= 12 - 7 \qquad \text{(subtract 7 from both sides)} \\ x &= 5 \end{aligned}$$

2.

$$\frac{x}{5} = 6$$

Your turn...

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1.

$$\begin{aligned} x + 7 &= 12 \\ x + 7 - 7 &= 12 - 7 && \text{(subtract 7 from both sides)} \\ x &= 5 \end{aligned}$$

2.

$$\begin{aligned} \frac{x}{5} &= 6 \\ \frac{x}{5} \times 5 &= 6 \times 5 && \text{(multiply both sides by 5)} \\ x &= 30 \end{aligned}$$

A really tricky question for you...

3.

$$\frac{x+5}{x} + 7 = 10$$

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A really tricky question for you. . .

3.

$$\frac{x+5}{x} + 7 = 10$$

$$\frac{x+5}{x} + 7 - 7 = 10 - 7 \qquad \text{(subtract 7 from both sides)}$$

$$\frac{x+5}{x} = 3$$

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$$\frac{x+5}{x} + 7 = 10$$

$$\frac{x+5}{x} + 7 - 7 = 10 - 7 \qquad \text{(subtract 7 from both sides)}$$

$$\frac{x+5}{x} = 3$$

$$\frac{x+5}{x} \times x = 3 \times x \qquad \text{(multiply both sides by } x\text{)}$$

$$x + 5 = 3x$$

A really tricky question for you...

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$$\frac{x+5}{x} = 3$$

$$\frac{x+5}{x} \times x = 3 \times x \qquad \text{(multiply both sides by } x\text{)}$$

$$x + 5 = 3x$$

$$x - x + 5 = 3x - x \qquad \text{(subtract } x \text{ from both sides)}$$

$$5 = 2x$$

A really tricky question for you...

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$$\frac{x+5}{x} + 7 = 10$$

$$\frac{x+5}{x} + 7 - 7 = 10 - 7 \qquad \text{(subtract 7 from both sides)}$$

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$$\frac{x+5}{x} \times x = 3 \times x \qquad \text{(multiply both sides by } x\text{)}$$

$$x + 5 = 3x$$

$$x - x + 5 = 3x - x \qquad \text{(subtract } x \text{ from both sides)}$$

$$5 = 2x$$

$$5 \times \frac{1}{2} = 2x \times \frac{1}{2} \qquad \text{(divide both sides by 2)}$$

$$\frac{5}{2} = x$$

Outline

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Two variables

Often we have two variables, y & x and two parameters a & b:

$$y = ax + b.$$

Definition (Linear function)

An equation with two variables of the form $y = ax + b$ is called a linear function.

▸ More

Definition (Independent and dependent variables)

The variable on the right hand side of the equation, x, is called the independent variable and the variable on the left hand side of the equation, y, is called the dependent variable.

- The dependent variable may also be written $y = f(x)$ or $y = g(x)$
- this notation emphasises that y is a function of x, in other words y depends on x.

▸ More

Graphing linear functions

- We use the cartesian plane:

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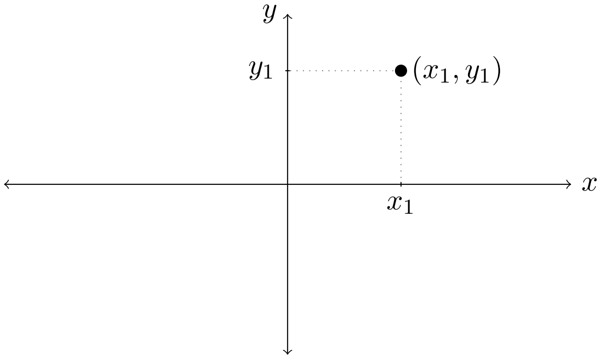
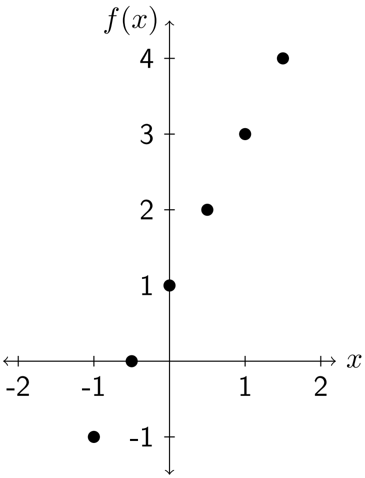


- When plotting a linear function, the independent variable is on the horizontal axis and the dependent variable is on the vertical axis.
- We refer to points on the cartesian plane as (x, y).

Graphing linear functions

One way to graph linear functions is to plot some points and join them. Consider the function, $f(x) = 2x + 1$:

x	$f(x) = 2x + 1$
-1	$2 \times (-1) + 1 = -1$
-0.5	$2 \times (-0.5) + 1 = 0$
0	$2 \times 0 + 1 = 1$
0.5	$2 \times 0.5 + 1 = 2$
1	$2 \times 1 + 1 = 3$
1.5	$2 \times 1.5 + 1 = 4$



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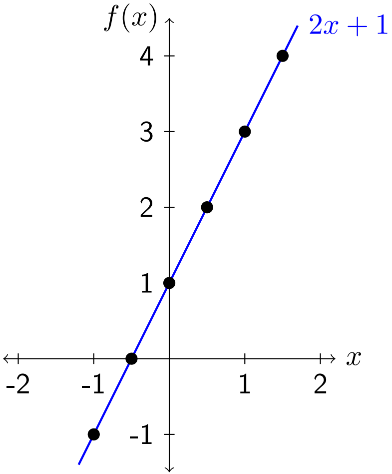

Gradient, slope, coefficient

Definition (Gradient)

In the linear function $y = ax + b$, the parameter a, that the variable x is multiplied by, is known as the gradient, slope or coefficient of x.

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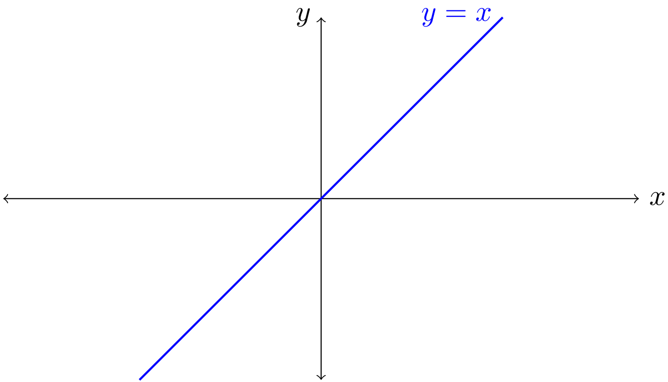

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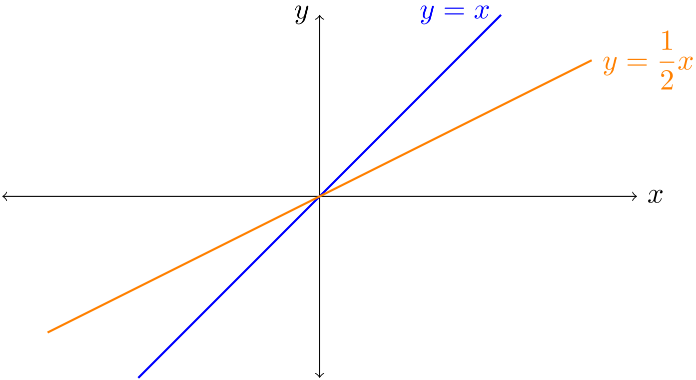

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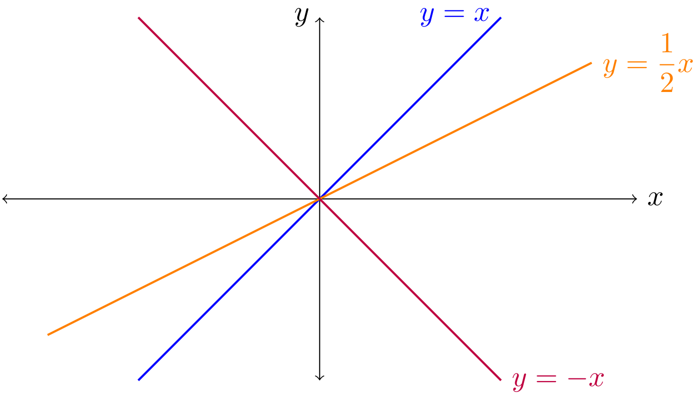

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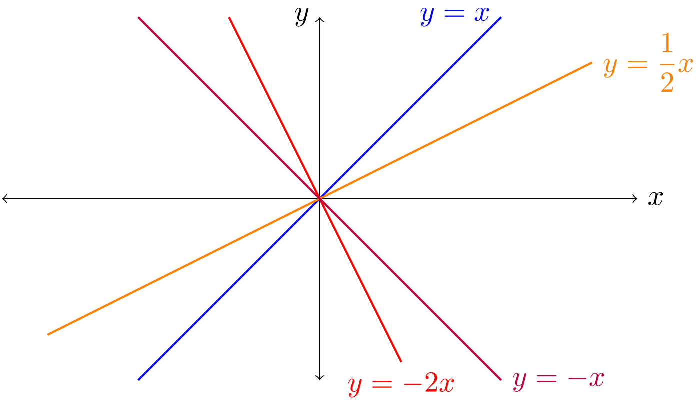

Intercept

Definition (Intercept)

In the linear function $y = ax + b$, when $x = 0$ this implies $y = b$. This means that b is the value of y at which the linear function crosses (or intercepts) the y axis.

- Hence, the parameter b is known as the intercept.

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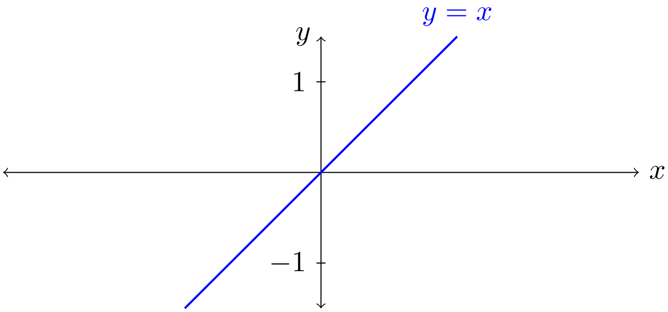

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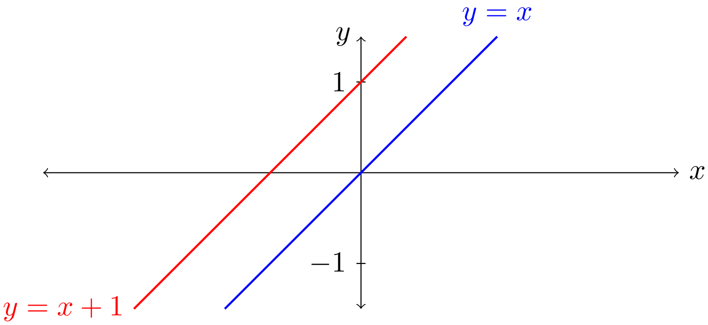

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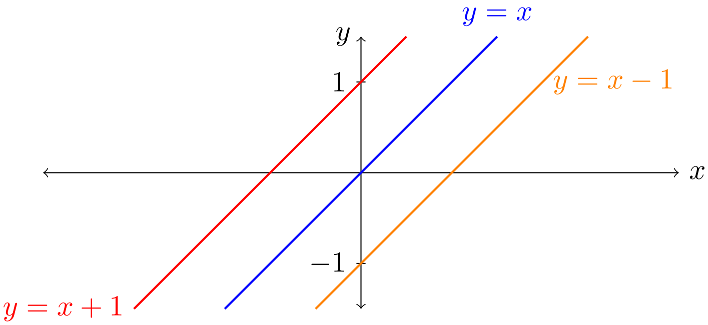
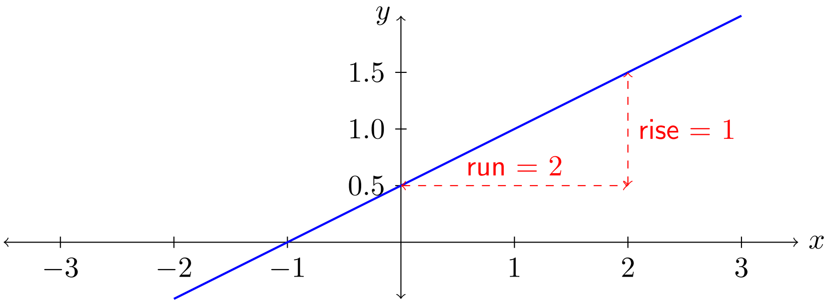


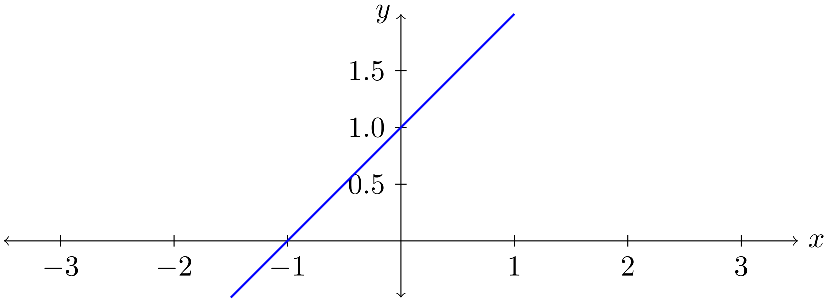
Given this line, find a and b in $y = ax + b$



Example

- When $x = 0$ we find the intercept: $b = 0.5$
- The slope is $a = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$
- The equation of the linear function is: $y = \dfrac{1}{2}x + \dfrac{1}{2}$

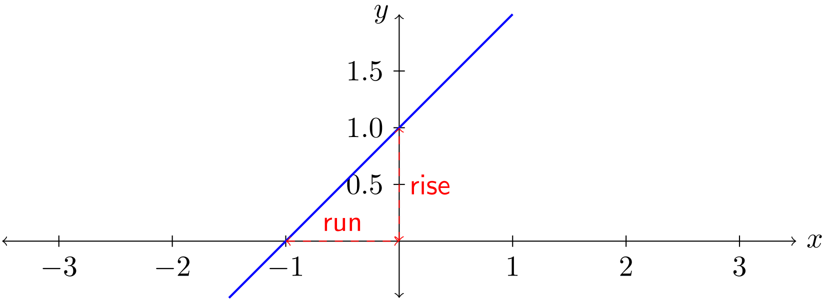
Your turn: find a and b in $y = ax + b$



Example (try yourself)

- When $x = 0$ we find the intercept: $b =$
- The slope is $a = \frac{\text{rise}}{\text{run}} =$
- The equation of the linear function is: $y =$

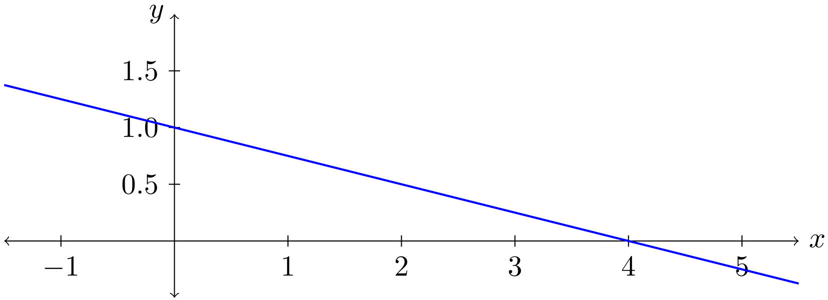
Your turn: find a and b in $y = ax + b$



Example (try yourself)

- When $x = 0$ we find the intercept: $b = 1$
- The slope is $a = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$
- The equation of the linear function is: $y = x + 1$

A little trickier: $y = ax + b$ with negative slope



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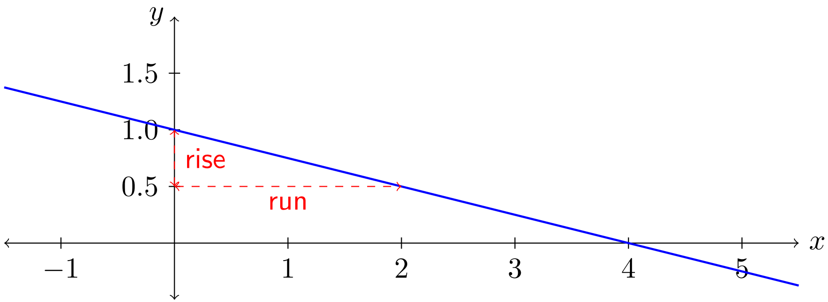


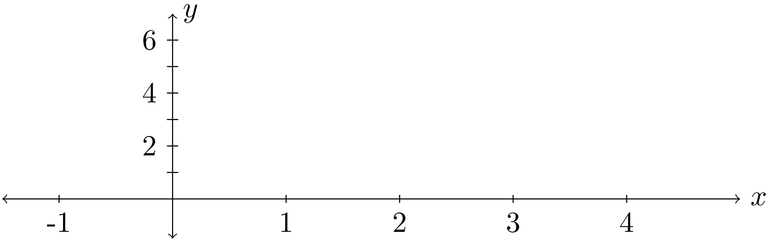
Example (try yourself)

- When $x = 0$ we find the intercept: $b = 1$
- The slope is $a = \frac{\text{rise}}{\text{run}} = -\frac{0.5}{2} = -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$
- The equation of the linear function is: $y = -\frac{1}{4}x + 1$

Plotting linear functions

Consider $y = -2x + 6$.

- The intercept is 6 and the slope is -2: could use this to draw the line.
- Often it is easier to find two points that the line passes through and draw the line through these two points.
    - When $x = 0$, $y = 6$.
    - When $y = 0 \implies 2x = 6 \implies x = 3$.
- The line passes through the two points (0,6) and (3,0)



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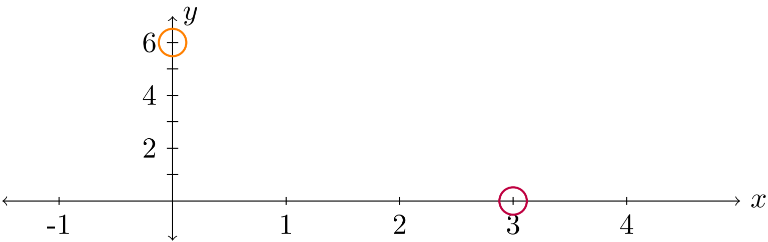

Plotting linear functions

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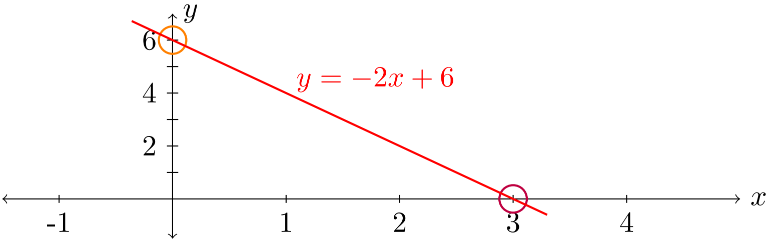
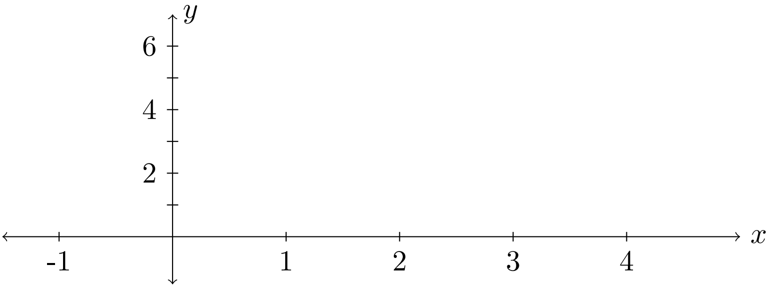


Your turn...

Plot the function

$$4x + 2y = 8$$

- Find two points that the line passes through:
  - $x = 0 \implies y =$
  - $y = 0 \implies x =$
- The line passes through the two points and



Your turn. . .

Plot the function

$$4x + 2y = 8$$

- Find two points that the line passes through:
  - $x = 0 \implies y = 4$
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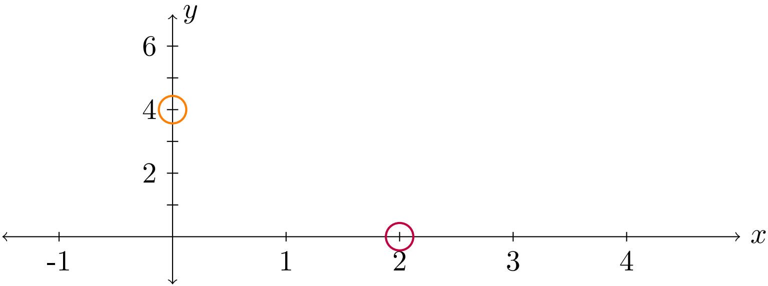
- The line passes through the two points (0,4) and (2,0)



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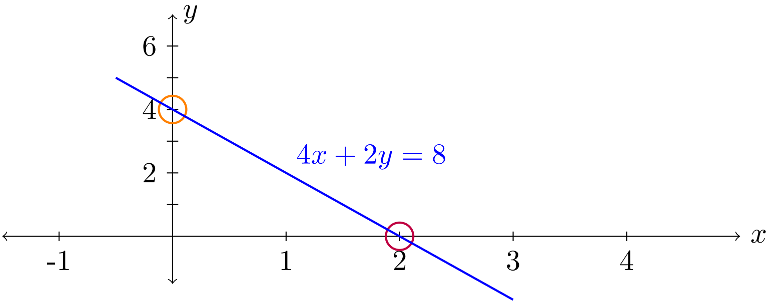


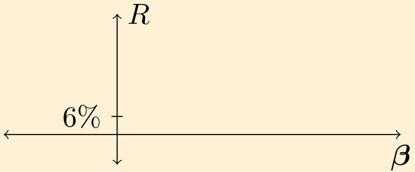
Application in Finance: CAPM

Capital Asset Pricing Model (CAPM)

The CAPM is a theoretical pricing model used in finance which predicts the return on an asset, R, to be linearly related to its sensitivity to the market, known as $\boldsymbol{\beta}$.

Example ($R = 6\% + 8\% \times \boldsymbol{\beta}$)

1. Graph this on the axes below (Hint: replace the usual x and y with $\boldsymbol{\beta}$ and R)
2. What is the return if the $\boldsymbol{\beta}$ of an asset is equal to 2?



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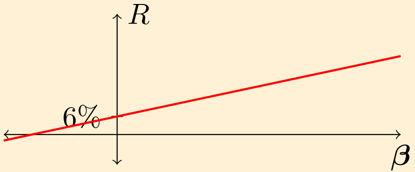

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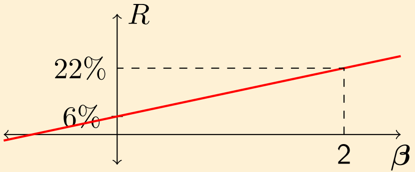

$$\begin{aligned} R &= 6\% + 8\% \times \boldsymbol{\beta} \\ &= 6\% + 8\% \times 2 \\ &= 22\% \end{aligned}$$

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Applications in Business

- In Finance the Capital Asset Pricing Model is a very popular linear function used to value an asset ▸ More
- In Accounting, depreciation is sometimes calculated using the "straight line" method ▸ More
- In Business Statistics simple linear regression fits a straight line through a data set ▸ More
- In Marketing the profitability of a strategy can often be summarised algebraically using a linear function with variables such as cost and response rate ▸ More

Outline

Definitions

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Definition (Sequence)

A sequence is an ordered list of objects (or events). For example,
$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots\right\}.$$

More

Definition (Series)

A series is the sum of the terms of a sequence. For example,
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$$

More

Definition (Limits)

A limit is the value that a sequence approaches as the input or index approaches some value. E.g. the limit of the sequence above as n approaches infinity is 0.

More

Arithmetic progression

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Definition (Arithmetic progression)

An arithmetic progression or arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant.

More

Example

The sequence 3, 5, 7, 9, 11, 13, . . . is an arithmetic progression with common difference 2.

In general any arithmetic sequence can be written as:

$$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, a_1 + 4d, \ldots, a_n, \ldots$$

- a_1 is the first term
- d is the common difference
- $a_n = a_1 + (n-1)d$ is the nth term in the sequence

Arithmetic series

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Definition (Arithmetic series)

The sum of an arithmetic progression is called an arithmetic series:

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_{n-1} + a_n.$$

More

We can find an explicit formula for S_n. Consider two different ways of expressing S_n,: (i) in terms of a_1; (ii) in terms of a_n

$$S_n = a_1 + (a_1 + d) + \ldots + (a_1 + (n-2)d) + (a_1 + (n-1)d)$$
$$S_n = (a_n - (n-1)d) + (a_n - (n-2)d) + \ldots + (a_n - d) + a_n$$

If we add the last two lines together, the terms involving d cancel out and we get:

$$2S_n = na_1 + na_n$$
$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(a_1 + [a_1 + (n-1)d]) = \frac{n}{2}(2a_1 + (n-1)d)$$

Arithmetic series

Example (Find the sum of the first 10 odd numbers)

The first 10 odd numbers are: $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

1. We can add the terms together using a calculator:

$$S_n = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$$

2. Or we can use the equation:

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{10}{2}(1 + 19) = 5 \times 20 = 100$$

Example (Find the sum of the first 100 odd numbers)

The first 100 odd numbers are: $\{1, 3, 5, \ldots, 197, 199\}$

1. It's not easy to do it manually so we use the equation:

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{100}{2}(1 + 199) = 50 \times 200 = 10,000$$

Arithmetic series

Example (Your turn...)

Your parents are setting up a trust fund that can give you \$1000 per year for every year while you are between the ages of 20 and 40 (inclusive) OR it can give you \$100 when you turn 20, \$200 when you turn 21, \$300 when you turn 22,... up until the final payment when you turn 40. Which option gives you more money in total assuming there's no inflation.

1. $n =$ so total is $S_n =$.
2. $a_1 =$,
   $a_n =$

   $S_n =$

   Therefore we prefer .

Arithmetic series

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Your parents are setting up a trust fund that can give you \$1000 per year for every year while you are between the ages of 20 and 40 (inclusive) OR it can give you \$100 when you turn 20, \$200 when you turn 21, \$300 when you turn 22,. . . up until the final payment when you turn 40. Which option gives you more money in total assuming there's no inflation.

1. $n = 21$ so total is $S_n = 1000 \times 21 = \$21,000$.
2. $a_1 = 100$,
   $a_n = a_{21} = a_1 + (n-1) \times d = 100 + 20 \times 100 = 2,100$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{21}{2}(100 + 2100) = \$23,100$$

Therefore we prefer option 2.

Geometric progression

Definition (Geometric progression)

A geometric progression or geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. More

Example

The sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3.

In general any geometric sequence can be written as:

$$a, ar, ar^2, ar^3, ar^4, \ldots, ar^{n-1}, ar^n, ar^{n+1}, \ldots$$

- a is the first term
- r is the common ratio

Geometric Series

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Definition (Geometric series)

The sum of a geometric progression is called a geometric series:

$$a + ar + ar^2 + \ldots + ar^{n-1} + ar^n = \sum_{k=0}^{n} ar^k.$$

▸ More

An explicit formula for the sum of the first $n+1$ terms:

- Let $s = 1 + r + r^2 + \ldots + r^{n-1} + r^n$
- Then $rs = r + r^2 + r^3 + \ldots + r^n + r^{n+1}$
- So $s - rs = (1 - r^{n+1})$ solving this for s we get:

$$s(1-r) = (1 - r^{n+1}) \implies s = \frac{1 - r^{n+1}}{1 - r}.$$

- If the start value is a, then we have:

$$\sum_{k=0}^{n} ar^k = a \times \frac{1 - r^{n+1}}{1 - r}.$$

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Limit of a geometric series

- We know that $\sum_{k=0}^{n} ar^k = \frac{a(1-r^{n+1})}{1-r}$.
- What happens as n approaches infinity? I.e. $n \to \infty$?
- If r is bigger than 1 or less than -1, i.e. $|r| > 1$, then r^n goes to either positive or negative infinity, i.e. $r^n \to \pm\infty$. E.g. $r = 2$ then $2^2 = 4, 2^3 = 8, 2^4 = 16 \ldots$ and the sum diverges.

More

- If r is between -1 and 1, i.e. $|r| < 1$, then r^n converges to zero, i.e. $r^n \to 0$ and so the sum becomes

$$\sum_{k=0}^{\infty} ar^k = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$$

and we say the sum converges.

More

Geometric series

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Example

An accountant's salary was $40,000 at the start of 1990. It increased by 5% at the beginning of each year thereafter. What was the accountant's salary at the beginning of 2010?

- At the beginning of 1990 it was $40,000$
- At the beginning of 1991 it was $40,000 \times (1 + 0.05) = 40,000 \times 1.05 = 42,000$
- At the beginning of 1992 it was $40,000 \times 1.05^2 = 44,100$
- At the beginning of 2010, $n = 20$ years time, it was $40,000 \times 1.05^{20} = 106,131.91$

What was the total amount earned over this period?

$$40000 \sum_{k=0}^{20} 1.05^k = 40000 \times \frac{1 - 1.05^{21}}{1 - 1.05} = \$1,428,770.$$

Compound interest

Definition (Compound interest)

Compound interest reflects interest that can be earned on interest

More

- We invest $\$A$ at the beginning of the first year, $t = 0$.
- At the end of the first year, $t = 1$, we have our initial investment plus the interest earned over the period:

$$A + rA = A(1 + r)$$

- At the end of the second year, $t = 2$, we have the amount from the start of the year plus interest:

$$A(1 + r) + A(1 + r)r = A(1 + r)(1 + r) = A(1 + r)^2$$

- At the end of the third year, $t = 3$, we have $A(1 + r)^3$
- Notice the pattern?
- The future value at time t is: $A\left(1 + r\right)^t$.

Compound interest

Example (Compound interest)

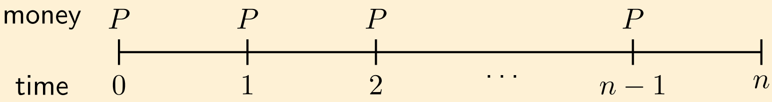
If \$1000 is invested at an interest rate of 10% per annum compounded annually, how much do you have at the end of 10 years?

- $A = 1000$
- $r = 0.1$
- $t = 10$
- $$A\,(1 + r)^t = 1000(1 + 0.1)^{10} = \$2593.74$$

Application: Superannuation

Example (Superannuation)

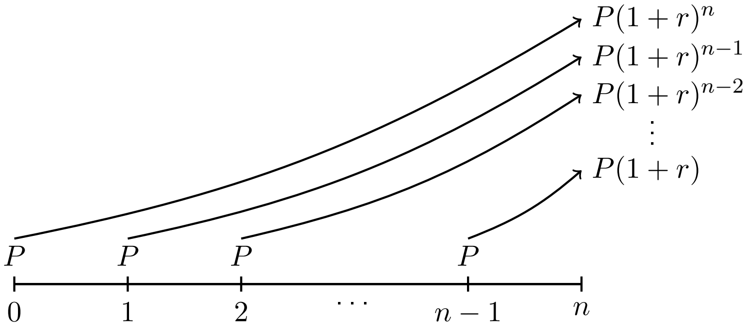
$\$P$ is invested at the start of every year for n years at a rate of $r\%$ per year.



- We want to know how much money we will have after n years with compound interest.

Application: Superannuation

If we think about each payment individually and consider its compound interest formula we have:



Therefore, the future value is the sum of all the components:

$$FV = P \times \left[(1+r) + \ldots + (1+r)^{n-2} + (1+r)^{n-1} + (1+r)^n\right]$$

Application: Superannuation

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We can re-express the present value formula using summation notation:

$$\begin{aligned} FV &= P(1+r) \times \left[1 + (1+r) + \ldots + (1+r)^{n-2} + (1+r)^{n-1}\right] \\ &= P(1+r) \times \sum_{k=0}^{n-1} (1+r)^k \end{aligned}$$

Which is a geometric sequence! Recall,

$$\sum_{k=0}^{n} c^k = \frac{1-c^{n+1}}{1-c} = \frac{c^{n+1}-1}{c-1}$$

So we have the future value:

$$FV = P(1+r) \times \frac{(1+r)^n - 1}{(1+r) - 1} = P(1+r) \times \frac{(1+r)^n - 1}{r}.$$

Application: Superannuation

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Example (Your turn...)

An investment banker pays \$10,000 into a superannuation fund for his mistress at the beginning of each year for 20 years. Compound interest is paid at 8% per annum on the investment. What will be the value at the end of 20 years?

$$FV = P(1+r) \times \frac{(1+r)^n - 1}{r}$$

- $P =$
- $r =$
- $n =$

$FV =$

Application: Superannuation

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Example (Your turn...)

An investment banker pays \$10,000 into a superannuation fund for his mistress at the beginning of each year for 20 years. Compound interest is paid at 8% per annum on the investment. What will be the value at the end of 20 years?

$$FV = P(1+r) \times \frac{(1+r)^n - 1}{r}$$

- $P = 10,000$
- $r = 0.08$
- $n = 20$

  $$FV = 10,000 \times (1+0.08) \times \frac{(1+0.08)^{20} - 1}{0.08} = \$494,229.20$$

- After 20 years it is worth almost half a million dollars!

Applications in Business

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Key concepts related to sequences and series

- Interest rates and the time value of money ▸ More
- Present Value ▸ More
- Future Value ▸ More
- Annuity ▸ More
- Perpetuity ▸ More

Primary uses in Business

- Amortisation in Accounting ▸ More
- Valuing cash flows in Finance ▸ More

Outline

Summary

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- parameters and variables
- substitution and solving equations
- dependent vs independent variable
- linear function: $f(x) = y = ax + b$
- slope (is it a parameter or a variable?)
- intercept (is it a parameter or a variable?)
- finding the equation for a linear function given a plot
- plotting a linear function given an equation
- Equations for solving sequences and series

Coming up...

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Week 4: Functions

- Understanding, solving and graphing Quadratic Functions
- Understanding Logarithmic and Exponential Functions

Week 5: Simultaneous Equations and Inequalities

- Algebraic and graphical solutions to simultaneous equations
- Understanding and solving inequalities

Week 6: Differentiation

- Theory and rules of Differentiation
- Differentiating various functions and application of Differentiation

Additional Resources

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- Test your knowledge at the University of Sydney Faculty of Economics and Business MathQuiz:
  `http://quiz.econ.usyd.edu.au/mathquiz`
- Additional resources on the Maths in Business website
  `sydney.edu.au/business/learning/students/maths`
- The University of Sydney Mathematics Learning Centre has a number of additional resources:
  - Maths Learning Centre algebra workshop notes ▸ More
  - Other Maths Learning Centre Resources ▸ More
- The Department of Mathematical Sciences and the Mathematics Learning Support Centre at Loughborough University have prepared a fantastic website full of excellent resources. ▸ More
- There's also tonnes of theory, worked questions and additional practice questions online. All you need to do is Google the topic you need more practice with! ▸ More

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- Questions, comments, feedback? Let us know at `business.maths@sydney.edu.au`