MATHS WORKSHOPS

Functions

Business School

THE UNIVERSITY OF SYDNEY

Outline

Outline

Functions

Definition (Function)

A function, f, is a *mapping* from one value, X, to another value, Z:

$$f : X \mapsto Z.$$

Think of the function, f, as a *machine* that takes an input, X, then transforms it in some way and outputs the result: Z.

More

Example ($f(x) = 2x - 3$)

The function $f(x) = 2x - 3$ has the following mapping:

- $f(3) = 2 \times 3 - 3 = 3$ so $f(x)$ maps 3 to 3.
- $f(2) = 2 \times 2 - 3 = 1$ so $f(x)$ maps 2 to 1.
- $f(1) = 2 \times 1 - 3 = -1$ so $f(x)$ maps 1 to -1.
- $f(0) = 2 \times 0 - 3 = -3$ so $f(x)$ maps 0 to -3.

Functions

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Think of the function, f, as a *machine* that takes an input, X, then transforms it in some way and outputs the result: Z. ▸ More

Example (Where have we seen functions before?)

We've been working with functions already:

- Linear Functions: $f(x) = ax + b$
- Quadratic Functions: $f(x) = ax^2 + bx + c$ (today's lesson)

- Sometimes functions are written as $y = f(x)$. For example you may see, $y = ax + b$ instead of $f(x) = ax + b$.

Outline

Quadratic Function

Definition (Quadratic Function)

A quadratic function takes the form:

$$f(x) = ax^2 + bx + c$$

where a, b and c are parameters and x is a variable.

More

Key point

The key point is that there is a term in the function involving the square of the variable, x^2.

Definition (Parabola)

The graph of a quadratic functions is a curve often referred to as a parabola.

More

Factorising quadratic function

Definition (Factorisation)

Quadratic functions can be factorised to take the form:

$$\begin{aligned} f(x) &= (x+d)(x+e) && \text{(factorised)} \\ &= (x+d)x + (x+d)e \\ &= x^2 + dx + xe + de \\ &= x^2 + (d+e)x + de && \text{(expanded)} \end{aligned}$$

▸ More

Example (Expand the following factorised quadratic)

$$(x-2)(x+1) =$$

Factorising quadratic function

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More

Example (Expand the following factorised quadratic)

$$(x-2)(x+1) = (x-2)x + (x-2) \times 1$$

Factorising quadratic function

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Definition (Factorisation)

Quadratic functions can be factorised to take the form:

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▸ More

Example (Expand the following factorised quadratic)

$$\begin{aligned} (x-2)(x+1) &= (x-2)x + (x-2) \times 1 \\ &= x^2 - 2x + x - 2 \\ &= x^2 - x - 2 \end{aligned}$$

Factorising quadratic functions

Example (Factorise $f(x) = x^2 + 12x + 32$)

We factorise this to $(x + d)(x + e)$ by matching the expansion:

$$x^2 + (d + e)x + de$$

with our example:

$$x^2 + 12x + 32.$$

I.e. we try to find factors d and e such that:

- P Their product is $de = 32$ so that the constants match.
- S Their sum is $d + e = 12$ so the coefficients of x match.
- F The factors are the two numbers whose sum is 12 and their product is 32. By trial and error we notice that $4 + 8 = 12$ and $4 \times 8 = 32$:

$$f(x) = (x + 4)(x + 8).$$

Factorising a more complex quadratic

The general form for a quadratic function is:

$$f(x) = ax^2 + bx + c$$

Example (Factorise $f(x) = 2x^2 + 3x - 5$)

Here we want to break up the middle term to help factorisation. To do this we find two numbers whose:

P product is $ac = 2 \times -5 = -10$

S sum is $b = 3$

F Using trial and error we find suitable candidates -2 and 5.

We can then re-write the original equation as:

$$\begin{aligned} 2x^2 + 3x - 5 &= 2x^2 - 2x + 5x - 5 \\ &= 2x(x-1) + 5(x-1) \\ &= (2x+5)(x-1) \end{aligned}$$

Your turn to factorise...

Factorise this expression

$$x^2 + 5x + 6$$

in the form $(x + d)(x + e)$.

P Product:

S Sum:

F Factors:

Your turn to factorise. . .

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Factorise this expression

$$x^2 + 5x + 6$$

in the form $(x + d)(x + e)$.

- P Product: Need two numbers that multiply to give 6
- S Sum: Need two numbers that sum to give 5
- F Factors:

Your turn to factorise...

Factorise this expression

$$x^2 + 5x + 6$$

in the form $(x + d)(x + e)$.

- P Product: Need two numbers that multiply to give 6
- S Sum: Need two numbers that sum to give 5
- F Factors: $2 + 3 = 5$ and $2 \times 3 = 6$

Your turn to factorise. . .

Factorise this expression

$$x^2 + 5x + 6$$

in the form $(x + d)(x + e)$.

P Product: Need two numbers that multiply to give 6

S Sum: Need two numbers that sum to give 5

F Factors: $2 + 3 = 5$ and $2 \times 3 = 6$

Therefore the factorisation is:

$$(x + 2)(x + 3)$$

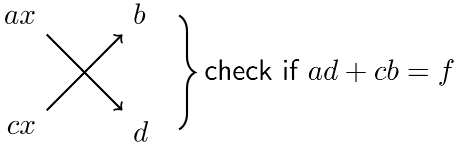
- You can verify this by expanding it out again!

Factorising using the cross method

In general, we can factorised $ex^2 + fx + g$ using

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

by finding a, b, c, d such that $ac = e$, $bd = g$ and $ad + bc = f$, using the cross method:



1. Pick an a and a c such that $ac = e$
2. Pick a b and a d such that $bd = g$
3. If $ad + cb = f$ then you have the solution.
   If $ad + cb \neq f$ go back to Step 1.

Your turn to factorise with a harder example. . .

$$2x^2 + 11x + 5$$

Method 1: Using a PSF-type approach:

- P Product:
- S Sum:
- F Factors:

Your turn to factorise with a harder example...

$$2x^2 + 11x + 5$$

Method 1: Using a PSF-type approach:

- P Product: Need two numbers that multiply to give $2 \times 5 = 10$
- S Sum: Need two numbers that sum to give 11
- F Factors:

Your turn to factorise with a harder example...

$$2x^2 + 11x + 5$$

Method 1: Using a PSF-type approach:

- P Product: Need two numbers that multiply to give $2 \times 5 = 10$
- S Sum: Need two numbers that sum to give 11
- F Factors: $10 + 1 = 11$ and $10 \times 1 = 10$

Your turn to factorise with a harder example. . .

$$2x^2 + 11x + 5$$

Method 1: Using a PSF-type approach:

- P Product: Need two numbers that multiply to give $2 \times 5 = 10$
- S Sum: Need two numbers that sum to give 11
- F Factors: $10 + 1 = 11$ and $10 \times 1 = 10$

We can use this to re-write the original expression:

$$\begin{aligned} 2x^2 + 11x + 5 &= 2x^2 + x + 10x + 5 \\ &= x(2x+1) + 5(2x+1) \\ &= (x+5)(2x+1). \end{aligned}$$

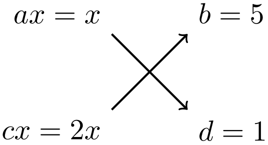
Your turn to factorise with a harder example. . .

$$2x^2 + 11x + 5$$

Method 2: the cross method to factorise this as:

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Here we want to find a, b, c, d such that $ac = 2$, $bd = 5$ and $ad + bc = 11$.

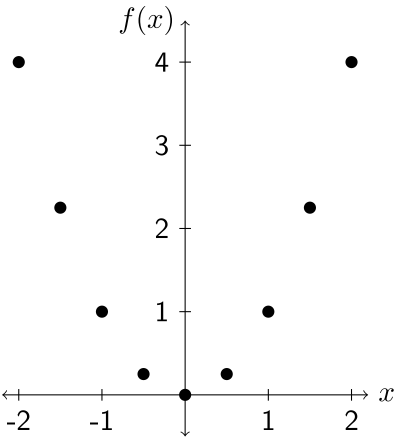


1. Pick $a = 1$ and $c = 2$ such that $ac = 2$
2. Pick $b = 5$ and $d = 1$ such that $bd = 5$
3. Check if $ad + cb = 11$. Here $1 \times 1 + 5 \times 2 = 11$, so we have a solution: $(x + 5)(2x + 1)$

Graphing quadratic functions

One way to graph quadratic functions would be to plot some points and join them. Consider the function, $f(x) = x^2$:

x	$f(x) = x^2$
-2	$(-2)^2 = 4$
-1.5	$(-1.5)^2 = 2.25$
-1	$(-1)^2 = 1$
-0.5	$(-0.5)^2 = 0.25$
0	$0^2 = 0$
0.5	$(0.5)^2 = 0.25$
1	$1^2 = 1$
1.5	$(1.5)^2 = 2.25$
2	$2^2 = 4$



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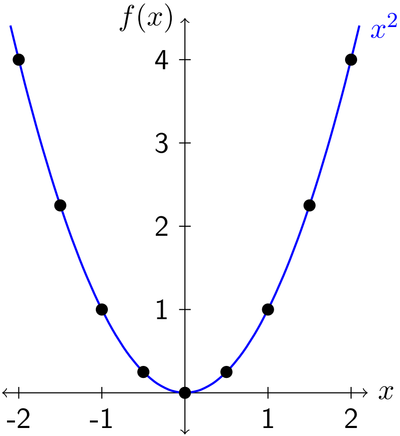
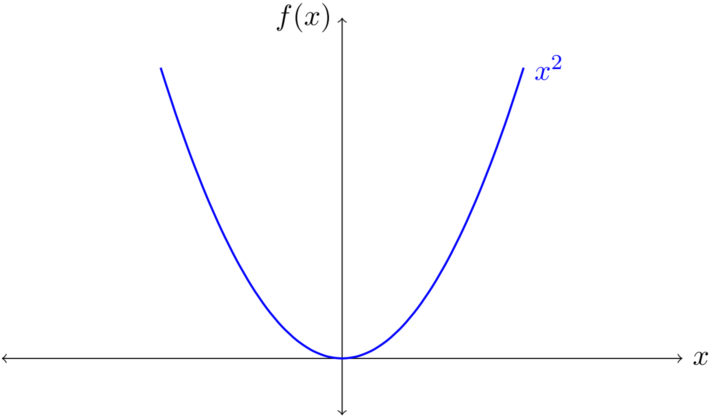


Graphing quadratic functions

Consider quadratic functions of the form $y = ax^2$.

- What does a do?



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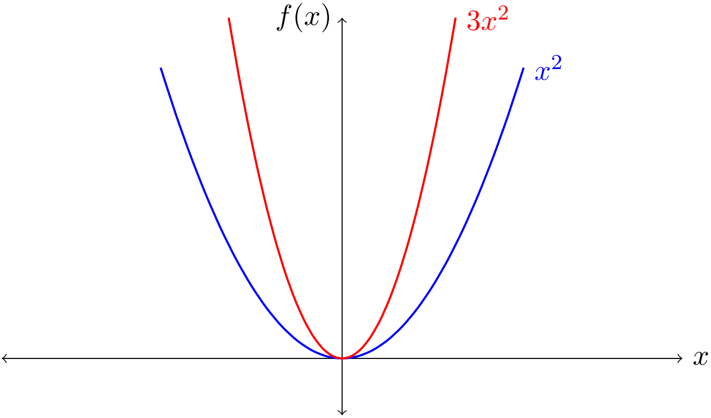

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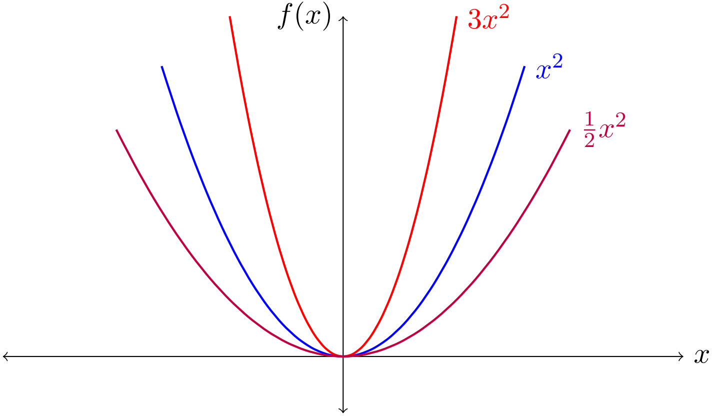
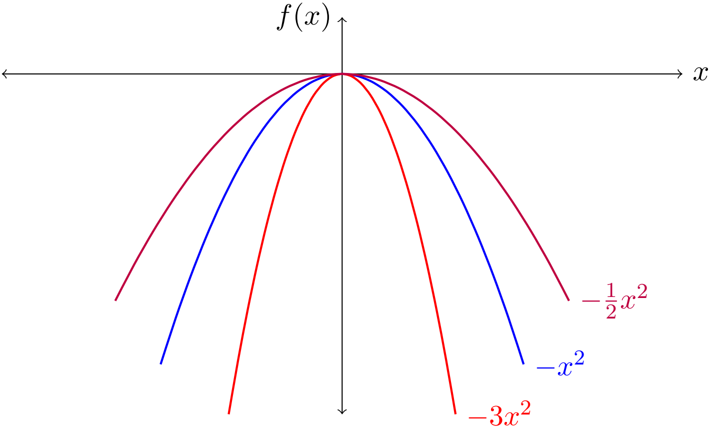


- a changes the "steepness" of the curve.

Graphing quadratic functions

Consider quadratic functions of the form $y = ax^2$.



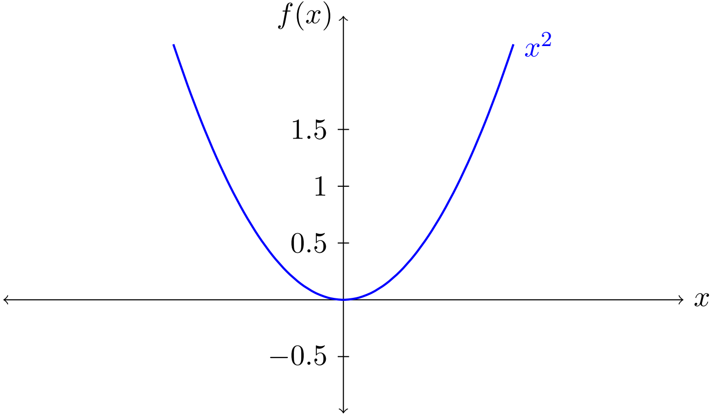
- The sign of a determines whether the parabola is convex (smile) or concave (frown).

More

Graphing quadratic functions

Consider quadratic functions of the form $f(x) = x^2 + c$.

- What does c do?



Graphing quadratic functions

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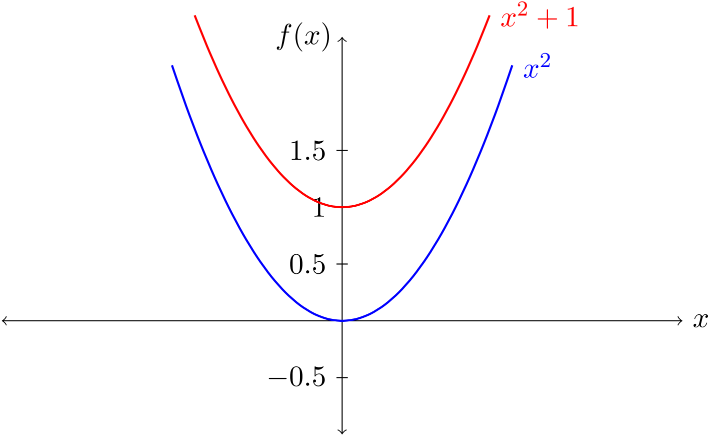

Graphing quadratic functions

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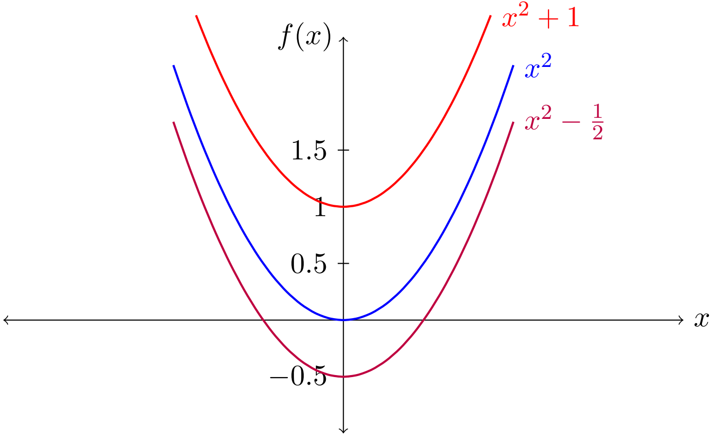
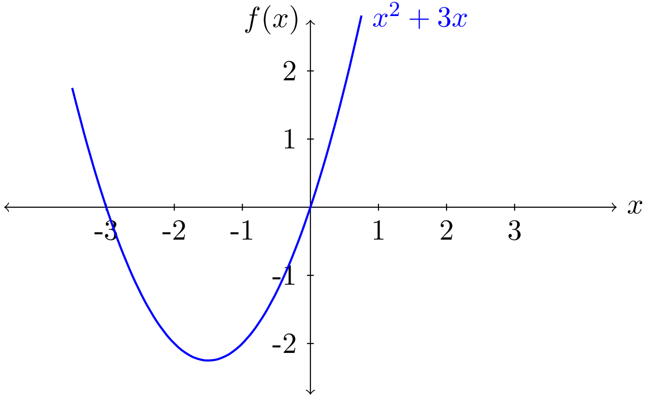


- c moves the curve up and down.

Graphing quadratic functions

Consider quadratic functions of the form $f(x) = ax^2 + bx + c$

- What does b do?



Graphing quadratic functions

Consider quadratic functions of the form $f(x) = ax^2 + bx + c$

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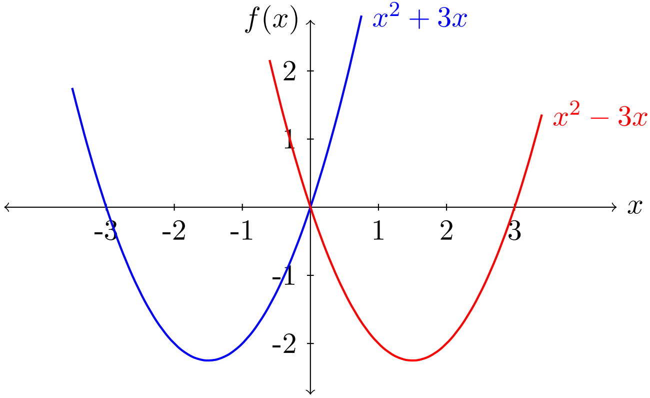

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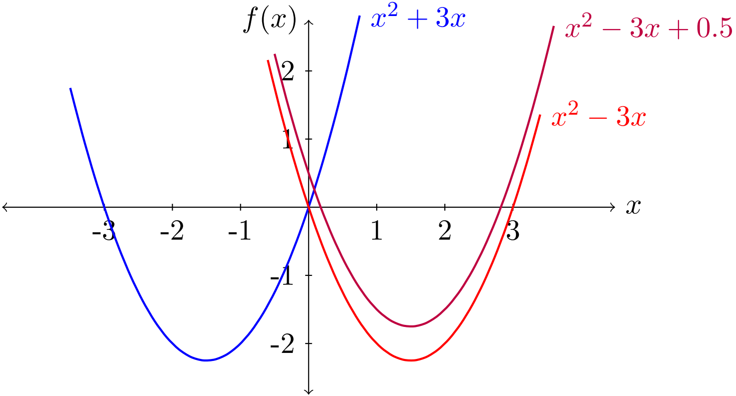
Consider quadratic functions of the form $f(x) = ax^2 + bx + c$

- What does b do?



- b moves the curve from side to side

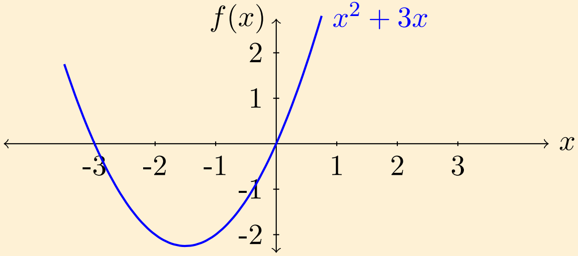
Finding the roots graphically

Definition (Roots)

The point(s) at which the quadratic function crosses the x axis are called the roots of the function.

More

Example (Finding the roots of $f(x) = x^2 + 3x$ graphically)



Finding the roots graphically

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More

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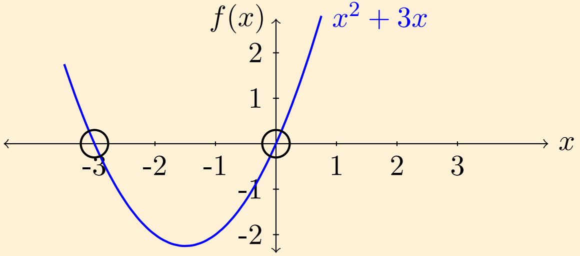


The roots occur at $x = -3$ and $x = 0$.

Finding the roots algebraically

- The definition says that the roots are "The point(s) at which the quadratic function crosses the x axis."
- Mathematically this is when $f(x) = 0$.
- This is easiest to find using the factorised form of $f(x)$.

Example (Finding the roots of $f(x) = x^2 + 3x$)

1. Factorise $f(x)$:

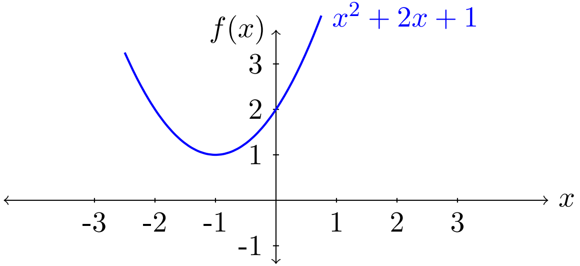
$$\begin{aligned} f(x) &= x^2 + 3x \\ &= x(x+3). \end{aligned}$$

2. Work out the values of x for which $f(x) = 0$ is true.
   - When $x = 0$ then $x(x+3) = 0$.
   - When $x = -3$ then $x(x+3) = 0$.

Therefore the roots are $x = 0$ and $x = -3$.

What if there aren't any roots?

The function $x^2 + 2x + 1$ doesn't cross the x axis at all!



Definition (Discriminant)

The function $f(x) = ax^2 + bx + c$ will only have root(s) if

$$b^2 - 4ac \geq 0.$$

- $b^2 - 4ac$ is known as the discriminant.

More

The Quadratic Formula

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Definition (Quadratic Formula)

If the quadratic function $f(x) = ax^2 + bx + c$ has roots, they can always be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

More

- Note that the square root of the discriminant is in the quadratic formula.
- This result suggests why there are no real roots unless $b^2 - 4ac \geq 0$. You cannot take the square root of a negative number.[1]

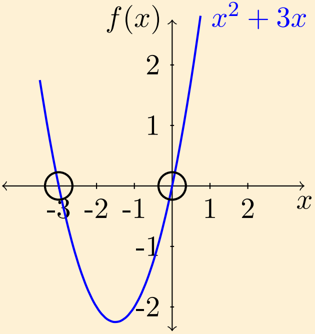
[1] You actually can but the solution is an imaginary number! More

The Quadratic Formula

Example (Finding the roots of $f(x) = x^2 + 3x$)

Here $a = 1$, $b = 3$ and $c = 0$ so

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 0}}{2 \times 1} \\ &= \frac{-3 \pm \sqrt{9 - 0}}{2} \\ &= \frac{-3 \pm 3}{2} \\ &= \frac{-3 + 3}{2} \quad \text{AND} \quad \frac{-3 - 3}{2} \\ &= 0 \quad \text{AND} \quad -3. \end{aligned}$$



Outline

Other common functions

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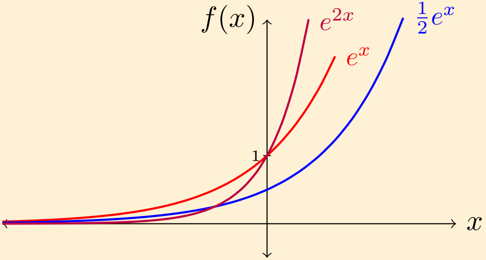
Example (Exponential function)

$$f(x) = ae^{cx+b}$$

where a, b, c and e are constants



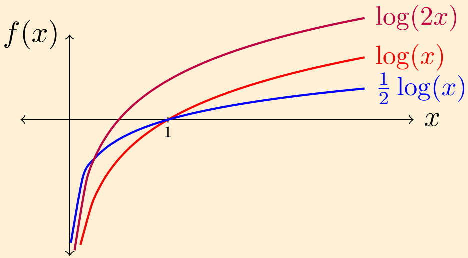
- The graph of $f(x) = e^x$ is upward-sloping, and increases faster as x increases. ▸ More
- The graph is always above the x-axis but gets arbitrarily close to it for negative x: the x-axis is an asymptote. ▸ More

Other common functions

Example (Logarithmic function)

$$f(x) = a\log(cx+b)$$

where a, b and c are constants



- The graph of $f(x) = \log(x)$ slowly grows to positive infinity as x increases. More
- The graph is always to the right of the y-axis but gets arbitrarily close to it for small x: the y-axis is an asymptote.

Relationship between logs and exponentials

- In $y = \log_a(b)$, a is known as the base of the log.
- We can change the base of the log using the relationship:

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}.$$

- Using this relationship, it is clear that:

$$\log_a(a) = \frac{\log_c(a)}{\log_c(a)} = 1.$$

- If $y = \log_a(x)$ then $x = a^y$
- Equivalently, if $y = a^x$ then $x = \log_a y$.
- If the base is the same, then the log function is the inverse of the exponential function:

$$a^{\log_a(x)} = x \quad \text{just like} \quad \frac{ax}{a} = x.$$

Natural Logs

- We typically work with log base $e \approx 2.7182818\ldots$.
- $\log_e(x)$ is often written as $\ln(x)$.
- If the base is left off the log, it's assumed $\log(x) = \log_e(x)$ (unless it's on your calculator, in which case it means $\log_{10}$)
- Note that the usual relations hold:

$$\begin{aligned} y &= e^x \\ \ln(y) &= \ln(e^x) && \text{(taking } \log_e \text{ of both sides)} \\ \ln(y) &= x \end{aligned}$$

- Also,

$$\begin{aligned} y &= \ln(x) \\ e^y &= e^{\ln(x)} && \text{(exponentiating both sides)} \\ e^y &= x \end{aligned}$$

Exponential and Log rules

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Exponentiation:

More

- $a^0 = 1$ for all $a \neq 0$.
- $a^{-1} = \frac{1}{a}$
- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

Logarithms:

More

- $\log(x^y) = y \log(x)$
- $\log(xy) = \log(x) + \log(y)$
- $\log\left(\frac{x}{y}\right) = \log(xy^{-1}) = \log(x) + \log(y^{-1}) = \log(x) - \log(y)$
- $\log(1) = 0$

Your Turn . . .

Solve the following equations for x

1. $\ln(x) = 2 \implies x =$
2. $\log_2 \frac{y}{3} = 4 \implies y =$

Simplify the following expressions

1. $e^{\ln 5} =$
2. $\ln \sqrt{e} =$
3. $e^{x + \ln x} =$
4. $\ln(1 + x) - \ln(1 - x) =$
5. $\dfrac{\ln(1 + x)}{\ln(e^2)} =$
6. $\log_3 3^q =$

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Your Turn ...

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Solve the following equations for x

1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

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Simplify the following expressions

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2. $\ln\sqrt{e} = \ln e^{1/2} = \frac{1}{2}\ln e = \frac{1}{2}$
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4. $\ln(1+x) - \ln(1-x) =$
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Simplify the following expressions

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2. $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}$
3. $e^{x + \ln x} = e^x e^{\ln x} = e^x x$
4. $\ln(1 + x) - \ln(1 - x) =$
5. $\dfrac{\ln(1 + x)}{\ln(e^2)} =$
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Your Turn . . .

Solve the following equations for x

1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

Simplify the following expressions

1. $e^{\ln 5} = 5$
2. $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}$
3. $e^{x + \ln x} = e^x e^{\ln x} = e^x x$
4. $\ln(1 + x) - \ln(1 - x) = \ln \left(\dfrac{1 + x}{1 - x} \right)$
5. $\dfrac{\ln(1 + x)}{\ln(e^2)} =$
6. $\log_3 3^q =$

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6. $\log_3 3^q = q \log_3 3 = q$

Solve the following equations for x

1. $$\ln(2x+1) = \ln(10-x)$$

2. $$2^{3x+1} = 4^x$$

3. $$\ln(2x+3) = 3$$

4. $$5^{x+1} = 200$$

Solve the following equations for x

1.
$$\begin{aligned}\ln(2x+1) &= \ln(10-x)\\ \exp\{\ln(2x+1)\} &= \exp\{\ln(10-x)\}\\ 2x+1 &= 10-x\\ x &= 3\end{aligned}$$

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4.
$$\begin{aligned} 5^{x+1} &= 200 \\ \ln(5^{x+1}) &= \ln(200) \\ (x+1)\ln(5) &= \ln(200) \\ x+1 &= \frac{\ln(200)}{\ln(5)} \\ x &= \frac{\ln(200)}{\ln(5)} - 1 \end{aligned}$$

Outline

Summary

- parameters and variables
- substitution and solving equations
- dependent vs independent variable
- linear function: $f(x) = y = ax + b$
- slope (is it a parameter or a variable?)
- intercept (is it a parameter or a variable?)
- finding the equation for a linear function given a plot
- plotting a linear function given an equation
- recognising quadratic functions
- factorising and expanding quadratics
- finding the roots of a quadratic using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- graphs of quadratic equations (parabolas)

Coming up. . .

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Week 5: Simultaneous Equations and Inequalities

- Algebraic and graphical solutions to simultaneous equations
- Understanding and solving inequalities

Week 6: Differentiation

- Theory and rules of Differentiation
- Differentiating various functions and application of Differentiation

Additional Resources

- Test your knowledge at the University of Sydney Business School MathQuiz: `http://quiz.econ.usyd.edu.au/mathquiz`
- Additional resources on the Maths in Business website `sydney.edu.au/business/learning/students/maths`
- The University of Sydney Mathematics Learning Centre has a number of additional resources:
  - Basic concepts in probability notes ▸ More
  - Sigma notation notes ▸ More
  - Permutations and combinations notes ▸ More
- There's also tonnes of theory, worked questions and additional practice questions online. All you need to do is Google the topic you need more practice with! ▸ More

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- Questions, comments, feedback? Let us know at `business.maths@sydney.edu.au`